

Syllabus for M.Tech. Online (AI) Written Test.

1. **Calculus:** Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.
2. **Linear Algebra:** Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.
3. **Probability:** Random variables. Uniform, normal, exponential, Poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem.

Sample Written Test: Please see the next few pages for a sample written test. Please note that no model answers will be provided.

Name: _____ Application Number: _____

Instructions. You will have 60 minutes to answer the questions. This paper has ten questions. For multiple choice questions, you must tick all the correct choices to get credit.

1. (1 point) The maximum of xe^{-x} is reached at $x^* =$ _____.
2. (1 point) The limit of the function $f(x) = \frac{\sin(\pi x)}{x}$ as x tends to 0 is equal to _____.
3. (1 point) The function $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined as

$$f(x) := \begin{cases} x^2 \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0, \end{cases}$$

where \mathbb{R} is the set of real numbers. Which all of the following hold?

[You must clearly tick all that apply, to get credit.]

- f is continuous for all $x \in \mathbb{R}$.
 - f is differentiable for all $x \in \mathbb{R}$.
 - f is differentiable for all $x \in \mathbb{R}$, and the derivative is continuous for all $x \in \mathbb{R}$.
 - f is differentiable for all $x \in \mathbb{R}$, and the derivative equals 0 at an infinite number of points.
4. (1 point) The sum of the squares of the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 4 \\ 4 & 5 & 1 \end{bmatrix}$$

is _____.

5. (1 point) The eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}$$

are (write your answer in the space given below):

6. (1 point) Which of the following choices hold true for the vectors $\left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \end{bmatrix} \right\}$
(You must tick all that apply.)

- Linearly independent
 Linearly dependent
 Neither linearly independent nor linearly dependent
 Orthogonal

7. (1 point) The rank of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

is _____.

8. (1 point) Let a biased coin be tossed n times in succession, with the probability of heads being p . The probability that all the tosses show the same face is _____.

9. (1 point) Let X and Y be two correlated random variables with means μ_X and μ_Y , respectively. The mean of the random variable $X + Y$ is always

- greater than $\mu_X + \mu_Y$
 less than $\mu_X + \mu_Y$
 $\frac{1}{2}\mu_X + \frac{1}{2}\mu_Y$
 $\mu_X + \mu_Y$

(You must tick all that apply.)

10. (1 point) A box contains 10 blue balls and 15 green balls. In the first draw, one ball is picked at random and discarded without noticing its colour. The probability that the second draw is green is _____.